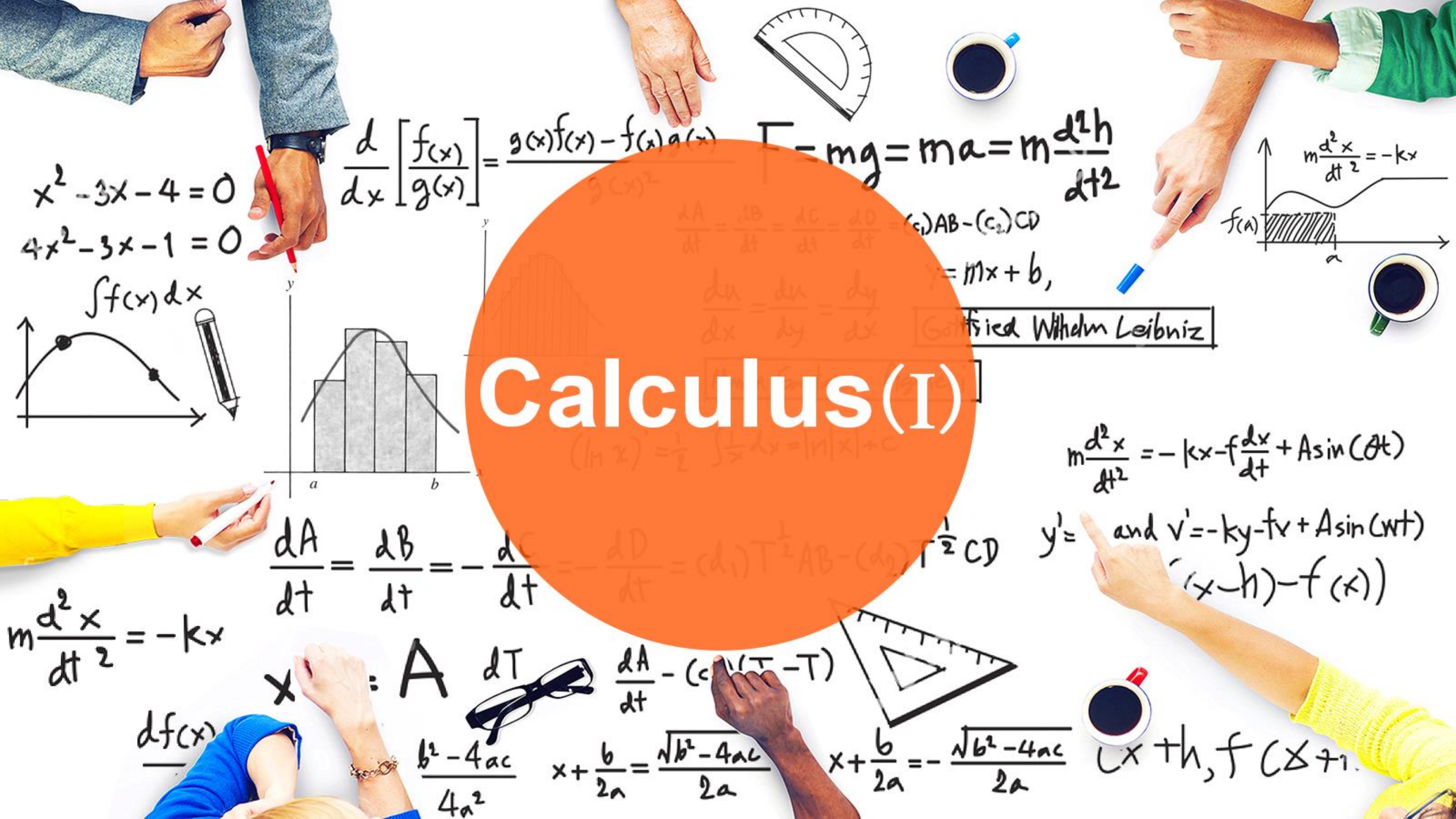
Calculus(Ⅰ)



Limits at Infinity; Infinite Limits

Lecturer: Xue Deng

Asymptote (Textbook P80+P82 Q49)

Def

P which belongs to $y = f(x)$ tends to infinity

If the distance of P and a fixed straight line L

L is called an asymptote of the curve $y = f(x)$.

How many cases?

Three (vertical, horizontal, oblique)

1. Vertical asymptote(vertical to x-axis)

The line $x = c$ is a vertival asymptote of the graph of $y = f(x)$ if any of the following four statements is true.

1. $\lim_{x \to c^+} f(x) = +\infty$
2. $\lim_{x \to c^+} f(x) = -\infty$
3. $\lim_{x \to c^-} f(x) = +\infty$
4. $\lim_{x \to c^-} f(x) = -\infty$

$$y = \frac{1}{(x+2)(x-3)}, \quad \lim_{x \to -2} \frac{1}{(x+2)(x-3)} = \infty$$

$$\lim_{x \to 3} \frac{1}{(x+2)(x-3)} = \infty$$

Vertical asymptote: $x = -2, \quad x = 3.$

2. Horizontal asymptote(parallel to x-axis)

The line $y = b$ is a horizontal asymptote of the graph of $y = f(x)$ if

$$\lim_{x \to +\infty} f(x) = b \quad \text{or} \quad \lim_{x \to -\infty} f(x) = b \quad (b \text{ is a constant})$$

$y = \arctan x$,

$$\lim_{x \to +\infty} \arctan x = \frac{\pi}{2} \qquad \lim_{x \to -\infty} \arctan x = -\frac{\pi}{2}$$

Horizontal asymptote: $y = \frac{\pi}{2}, \quad y = -\frac{\pi}{2}.$

3. Oblique asymptote

The line $y = ax + b$ is called an oblique asymptote

to the graph of $y = f(x)$ if

$$\lim_{x \to \pm\infty} [f(x) - (ax + b)] = 0 \qquad (a, b \text{ are constants and } a \neq 0) \; (1)$$

Method of finding a and b?

Question

Method of finding a and b? $\lim_{x\to\pm\infty}[f(x)-(ax+b)]=0$

By (1) and x is infinity, have $\lim_{x\to\pm\infty}\frac{1}{x}[f(x)-(ax+b)]=0$

Namely, $\lim_{x\to\pm\infty}[\frac{f(x)}{x}-a-\frac{b}{x}]=\lim_{x\to\pm\infty}\frac{f(x)}{x}-a=0$

So $a=\lim_{x\to\pm\infty}\frac{f(x)}{x}$ a is obtained, from (1) to find b,

$$b=\lim_{x\to\pm\infty}[f(x)-ax]$$

Note

$$a = \lim_{x \to \pm\infty} \frac{f(x)}{x}, \quad b = \lim_{x \to \pm\infty} [f(x) - ax]$$

If

(1) $\lim\limits_{x \to \pm\infty} \frac{f(x)}{x}$ does not exist;

(2) $\lim\limits_{x \to \pm\infty} \frac{f(x)}{x} = a$ exists, but $\lim\limits_{x \to \pm\infty} [f(x) - ax]$ does not exist,

determie: no asymptote abouy $y = f(x)$.

Question

Find asymptote of $f(x) = \dfrac{2(x-2)(x+3)}{x-1}$.

Domain: $(-\infty, 1) \cup (1, +\infty)$.

1 $\because \lim_{x \to 1} f(x) = \infty$,

$\therefore x = 1$ vertical asymptote

2 $\because \lim_{x \to \infty} \dfrac{2(x-2)(x+3)}{x-1} = \infty$

So, no horizontal asymptote.

Question

Find asymptote of $f(x)=\dfrac{2(x-2)(x+3)}{x-1}$.

3

$$a=\lim_{x\to\pm\infty}\frac{f(x)}{x},\quad b=\lim_{x\to\pm\infty}[f(x)-ax]$$

And $\because \lim_{x\to\infty}\dfrac{f(x)}{x}=\lim_{x\to\infty}\dfrac{2(x-2)(x+3)}{x(x-1)}=2=a$

$$\lim_{x\to\infty}[f(x)-ax]=\lim_{x\to\infty}[f(x)-2x]$$

$$=\lim_{x\to\infty}[\frac{2(x-2)(x+3)}{x-1}-2x]$$

$$=\lim_{x\to\infty}\frac{2(x-2)(x+3)-2x(x-1)}{x-1}=4=b$$

$\therefore y=2x+4$ is an oplique asymptote.

Exercise

Selection Q:

Curve $y = \dfrac{x|x|}{(x-1)(x-2)}$, the number of asymptotes is

(A) 1. **(B)** 2. **(C)** 3. **(D)** 4.

Limits at Infinity; Infinite Limits